THE DE RHAM COHOMOLOGY OF p-GROUP COVERS DESCRIPTION FOR GENERAL PUBLIC (EXTENDED)

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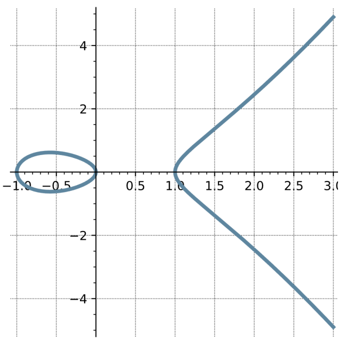
1. "Symmetries" of curves

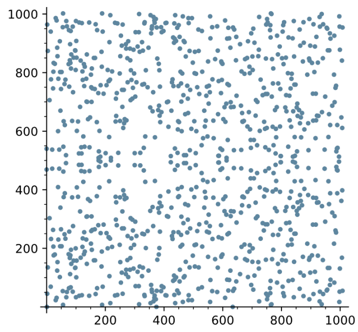
It is often the case in many areas of science, that we want to study not only an object, but also its symmetries, as they form an inherent part of its structure. Often this allows us to understand the object better. This philosophy comes up for example, when we consider a chemical molecule or a physical system. In mathematics this is formalized by using such notions as a **group**, **group action** and a **representation of a group**. The proposed project concerns symmetries of algebraic curves, i.e. one dimensional objects given by algebraic equations, e.g.

$$y^2 = x^3 - x. \tag{$*$}$$

Note however, that we will deal with curves modulo a prime p. This means that we want the equation to by satisfied only up to a multiple of p. For example, the point $(x, y) = (662, 578)$ belongs to the curve with the equation $(*)$ modulo 1009, since $578^2 = 662^3 - 662 + 287198 \cdot 1009$.



"Classical" curve $y^2 = x^3 - x$



Curve $y^2 = x^3 - x$ modulo 1009

In the left picture we see a classical curve, in the right – the curve with the same equation but modulo 1009. Note that both curves have a symmetry (and it is literally a symmetry with respect to a horizontal line).

Curves modulo p have a major importance in mathematics – for example the famous Riemann hypothesis, which is unsolved in the classical case, is known for curves modulo p. Also, curves modulo primes have numerous applications in cryptography: both to encrypt messages, to construct error-correcting codes and to provide digital signatures.

2. Cohomologies

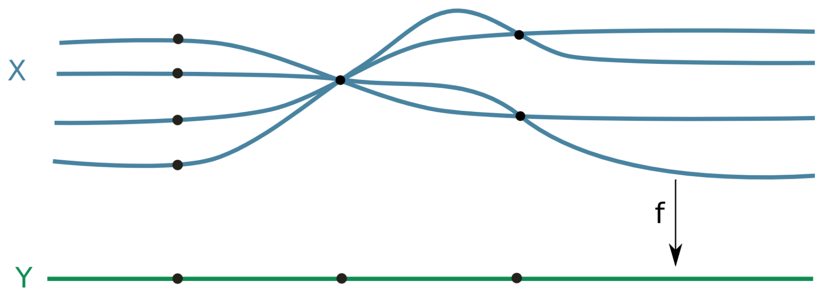
There are various invariants of curves that distinguish them. Let mention only two of them:

- the **de Rham cohomology**: $H^1_{dR}(X)$,
- the **Hodge cohomology**: $H^1_{Hdg}(X)$.

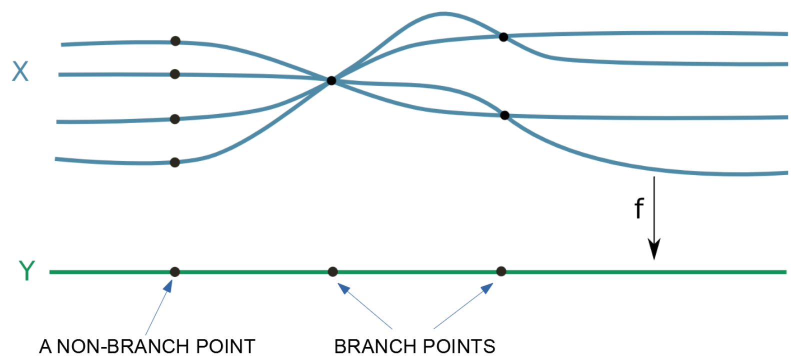
Both of them are certain vector spaces. For classical curves those two cohomologies are the same, for curves mod p they might not be. If the curve has some symmetries, then these symmetries act also on the cohomologies. The goal of this project is to understand those two invariants, accounting for the action of the symmetries.

3. Branch points

In order to explain a conjecture concerning the cohomologies, I must first define the notion of a **branch point**. Considering a curve X with a set of "symmetries" G is equivalent to considering a **cover of curves** $f : X \to Y$. Here Y is the space of "orbits", meaning that in order to obtain Y we identify the points of X that are symmetrical. In the picture we see an exemplary case, when there are four symmetries.



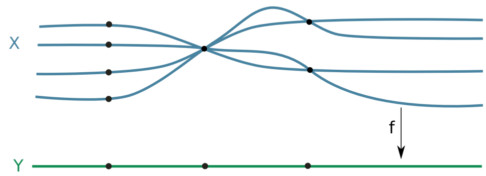
Note that over almost every point of Y there are four points of X. The remaining points are **branch points**. Similarly one defines branch points in general.



4. Conjecture

My previous results suggest that the de Rham cohomology decomposes as a sum of certain global and local parts. The global part should depend only on the "topology" of the cover (i.e. on the picture of the cover, as drawn above), while the local parts should depend only on a small neighbourhood of the branch points of the cover. Moreover, the global part of the Hodge cohomology and the de Rham cohomology should be the same. Thus, Hodge and de Rham cohomology differ only by local parts! I expect also that the local parts can be described in terms of Harbater–Katz–Gabber covers, i.e. covers of the line that are branched only over one point.

This conjecture has a nice geometric interpretation. Namely, it means that the cohomology of the cover:



is the same as of the degenerated cover approximating it: