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LOCAL TORSION

CM CURVES

SUPERSINGULA ELLIPTIC CURVES

Open problems

CLASS NUMBERS O ABELIAN VARIETIES

# LOCAL TORSION OF ELLIPTIC CURVES

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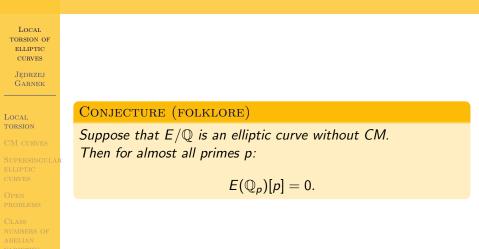
Adam Mickiewicz University, Poznan



Torsion groups and Galois representations of elliptic curves 25.06.2018

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# LOCAL TORSION PROBLEM



# *p*-DEGREE CONJECTURE

Local torsion of elliptic curves

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#### DEFINITION

*p*-degree of an elliptic curve  $E/\mathbb{Q}$ :

 $d_p(E) = \min\{[L:\mathbb{Q}_p]: E(L)[p] \neq 0\}$ 

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# *p*-DEGREE CONJECTURE

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 $d_p(E) = \min\{[L:\mathbb{Q}_p]: E(L)[p] \neq 0\}$ 

A more general conjecture:

CONJECTURE (DAVID & WESTON, 2008)

If  $E/\mathbb{Q}$  is an elliptic curve and End  $E = \mathbb{Z}$ , then:

 $\lim_{p\to\infty}d_p(E)=\infty.$ 

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# *p*-DEGREE CONJECTURE

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CLASS NUMBERS O ABELIAN VARIETIES DEFINITION

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Motivation: the deformation theory of Galois representations.

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CLASS NUMBERS C ABELIAN VARIETIES What happens for elliptic curves with CM?

THEOREM (J.G., 2018)

Let  $E: y^2 = x^3 - x$ . Then for any prime  $p \neq 2, 3$ :

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$$d_p(E) =$$

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CLASS NUMBERS C ABELIAN VARIETIES What happens for elliptic curves with CM?

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$$d_p(E) = \begin{cases} p^2 - 1, & \text{for } p \equiv 3 \pmod{4}, \\ \end{cases}$$

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where s is defined for  $p \equiv 1 \pmod{4}$  by  $p = s^2 + t^2$ 

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where s is defined for  $p \equiv 1 \pmod{4}$  by  $p = s^2 + t^2$  and

 $2 \nmid s, \qquad s+t \equiv 1 \pmod{4}.$ 

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Original proof: main theorem of complex multiplication.

# $\ensuremath{\textit{p}}\xspace$ of elliptic curves with CM

Local torsion of elliptic	Corollary (J.G., 2018)
CURVES	For
Jędrzej Garnek	$E: y^2 = x^3 - x$
CM CURVES	
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# COROLLARY (J.G., 2018)

For

$$E: y^2 = x^3 - x$$

we have

 $d_p(E) = 8$ 

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#### COROLLARY (J.G., 2018)

For

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we have

$$d_p(E) = 8$$

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if and only if p is of the form  $s_{k+1}^2 + s_k^2$ , where:

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# Corollary (J.G., 2018)

For

$$E: y^2 = x^3 - x$$

we have

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if and only if p is of the form  $s_{k+1}^2 + s_k^2$ , where:

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#### Remark

$$(s_{k+1}^2 + s_k^2)_{k=1}^{100\,000}$$
 is a prime iff

 $k \in \{1, 2, 3, 4, 5, 131, 200, 296, 350, 519, 704, 950, 5598,$ 

6683, 7445, 8775, 8786, 11565, 12483 }.

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$$d_p(E) = \begin{cases} p^2 - 1, & \text{for } p \equiv 3 \pmod{4}, \\ \text{ord}_p(2s), & \text{for } p \equiv 1 \pmod{4}, \end{cases}$$

$$(\text{for } E: y^2 = x^3 - x)$$

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Both parts of the formula

 $d_p(E) = \begin{cases} p^2 - 1, & \text{for } p \equiv 3 \pmod{4}, \\ \text{ord}_p(2s), & \text{for } p \equiv 1 \pmod{4}, \end{cases}$   $(\text{for } E: y^2 = x^3 - x)$ 

may be generalized!

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# *p*-DEGREE OF SUPERSINGULAR ELLIPTIC CURVES

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#### THEOREM (J.G., 2018)

If  $E/\mathbb{Q}_p$  has good supersingular reduction then

$$d_p(E)=p^2-1.$$

**Proof:** study of the formal group law of  $E \Rightarrow$  for any  $P \in E[p]$ :

$$e(\mathbb{Q}_p(P)/\mathbb{Q}_p) = p^2 - 1.$$

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# *p*-DEGREE OF SUPERSINGULAR ELLIPTIC CURVES

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$$e(\mathbb{Q}_p(P)/\mathbb{Q}_p)=p^2-1.$$

By Elkies' results:

COROLLARY

For any elliptic curve  $E/\mathbb{Q}$ :

 $\limsup_{p\to\infty} d_p(E) = \infty.$ 

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#### DEFINITION

**Canonical lift** of an ordinary elliptic curve  $E/\mathbb{F}_q$ :

the only lift  $\mathbb{E}/\mathcal{W}(\mathbb{F}_q)$  of E with CM.

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The following conditions are "almost equivalent":

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### THEOREM (GROSS; DAVID & WESTON; GARNEK)

The following conditions are "almost equivalent": (1)  $d_p(E) ,$  $(2) <math>E_{\mathbb{F}_p}$  is ordinary and  $E_{\mathbb{Z}/p^2}$  is a canonical lift of  $E_{\mathbb{F}_p}$ ,

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The following conditions are "almost equivalent":

- (1)  $d_p(E) ,$
- (2)  $E_{\mathbb{F}_p}$  is ordinary and  $E_{\mathbb{Z}/p^2}$  is a canonical lift of  $E_{\mathbb{F}_p}$ ,
- (3)  $E(\mathbb{Q}_p^{un})[p] \neq 0$ ,

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(2)  $E_{\mathbb{F}_p}$  is ordinary and  $E_{\mathbb{Z}/p^2}$  is a canonical lift of  $E_{\mathbb{F}_p}$ ,

- (3)  $E(\mathbb{Q}_p^{un})[p] \neq 0$ ,
- (4)  $E_{\mathbb{F}_p}$  is ordinary and  $d_p(E) = \operatorname{ord}_p a_p(E)$ .

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 $(2)~{\it E}_{{\Bbb F}_p}$  is ordinary and  ${\it E}_{{\Bbb Z}/p^2}$  is a canonical lift of  ${\it E}_{{\Bbb F}_p},$ 

3) 
$$E(\mathbb{Q}_p^{un})[p] \neq 0$$
,

(4)  $E_{\mathbb{F}_p}$  is ordinary and  $d_p(E) = \operatorname{ord}_p a_p(E)$ .

Precisely, "almost equivalent" =

 $(1) \Rightarrow (2), (2) \Leftrightarrow (3), (3) \Rightarrow (4).$ 

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#### QUESTION

# What is the behaviour of $d_p(E)$ for $p \to \infty$ for other elliptic curves $E/\mathbb{Q}$ ?

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What is the behaviour of  $d_p(E)$  for  $p \to \infty$  for other elliptic curves  $E/\mathbb{Q}$ ?

#### QUESTION

How often is an elliptic curve  $E/\mathbb{Q}$  the canonical lift  $mod p^2$  of its reduction mod p?

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What about abelian varieties?

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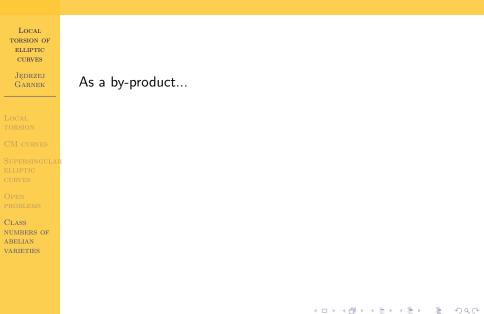
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What about abelian varieties?

#### QUESTION

Fix a Jacobian  $A/\mathbb{Q}$ . How often is the canonical lift of A mod p a Jacobian mod  $p^2$ ?

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CLASS NUMBERS OF ABELIAN VARIETIES As a by-product...

- $A/\mathbb{Q}$  an abelian variety of dimension d,
- r rank of  $A(\mathbb{Q})$  over  $\operatorname{End}_{\mathbb{Q}}(A)$ ,
- p a fixed prime number,
- $K_n := \mathbb{Q}(A[p^n]) p^n$ th division field of A

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# CLASS NUMBERS OF ABELIAN VARIETIES



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#### QUESTION

How to estimate the class number of  $K_n$ ?

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#### THEOREM (J.G., 2018)

If either of the following conditions holds:

• *r* > *d*,

•  $r \ge 1$ , A has good reduction at p and  $A_{\mathbb{F}_p}[p] \neq 0$ ,

then for some explicit C = C(A, p) > 0, D = D(A, p) > 0:

 $\# \operatorname{Cl}(K_n) \ge p^{Cn-D}.$ 

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Idea of the proof:

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# Idea of the proof:

• investigate the Kummer extension of  $\mathbb{Q}(A[p^n])$ ,

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#### THEOREM (J.G., 2018)

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then for some explicit C = C(A, p) > 0, D = D(A, p) > 0:

$$\# \operatorname{Cl}(K_n) \geqslant p^{Cn-D}$$

#### Idea of the proof:

- investigate the Kummer extension of  $\mathbb{Q}(A[p^n])$ ,
- switch to local extension to give a bound on inertia groups.

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On *p*-degree of elliptic curves International Journal of Number Theory, 2018.

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# Thank you for your attention!

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